Description Logics with I and me

Oberseminar – FAU Erlangen – WS2012/13

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November 13th, 2012

Last update: November 13, 2012

Consider a world with:

$$\begin{aligned} \text{Man} &\equiv \text{Person} \sqcap \text{Male} \\ \text{Woman} &\equiv \text{Person} \sqcap \text{Female} \\ \bot &\equiv \text{Male} \sqcap \text{Female} \\ \text{Father} &\sqsubseteq \text{Man} \sqcap \exists \text{parentOf.Person} \\ \text{Father} &\sqsubseteq \forall \text{parentOf.Person} \end{aligned}$$

So given parentOf(Anakin, Luke) and Father(Anakin) for this world implies:

Consider a world with:

$$
\begin{aligned}
\mathrm{Man} &\equiv \mathrm{Person} \sqcap \mathrm{Male} \\
\mathrm{Woman} &\equiv \mathrm{Person} \sqcap \mathrm{Female} \\
\bot &\equiv \mathrm{Male} \sqcap \mathrm{Female} \\
\mathrm{Father} &\sqsubseteq \mathrm{Man} \sqcap \exists \mathrm{parentOf}.\mathrm{Person} \\
\mathrm{Father} &\sqsubseteq \forall \mathrm{parentOf}.\mathrm{Person}
\end{aligned}
$$

So given parentOf(Anakin, Luke) and Father(Anakin) for this world implies:

- $\neg$ Female(Anakin)
- Person(Luke)
- Woman $\sqcap$ Father $\equiv \bot$

Definition of Description Logic

Family of knowledge representation languages, where each models:

- Concepts – Properties of an individual
- Roles – Relations between two individuals
- Assertions for roles and concepts: ABox
- Relations between concepts: TBox
- Relations between roles: RBox

Every Description Logic (e.g. $\mathcal{ALC}$, $\mathcal{ALCHIQ}$, $\mathcal{ALCHIQ}$me$_2$, ...) defines rules how these properties can be described.

[A DL Primer]

The Description Logic $\mathcal{ALC}$ allows:

Concepts

Compose new concepts using $\sqcup, \sqcap, \neg, \exists, \forall$

ABox

Assertions: Mother(Padme), parentOf(Padme, Leia)

TBox

Relation between concepts: Mother $\sqsubseteq$ Parent

RBox

None

[A DL Primer]

The Description Logic $\mathcal{ALCHIQ}$ additionally allows:

- Role $\mathcal{H}$ierarchies: parentOf $\sqsubseteq$ ancestorOf
- $\mathcal{I}$nverse Roles: childOf $:\equiv$ parentOf$^-$
- $\mathcal{Q}$ualified number restrictions: Parent $:\equiv \geq_1$ parentOf.Human

$$\text{Human} \sqsubseteq (\geq_2 \text{parentOf}^-.\text{Human})$$

[A DL Primer]

Model

A *non-empty* set Δ^I of individuals, where each

- concept C is represented by a subset: $C^I \subseteq \Delta^I$
- role R is represented by a relation: $R^I \subseteq \Delta^I \times \Delta^I$
- axiom holds.

$\mathcal{ALCQ}$me

- I remembers the current point of evaluation.
- me only matches the point marked by I.

[Marx]

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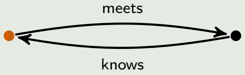
Narcissist

$\text{I}.\exists\text{loves}.\text{me}$

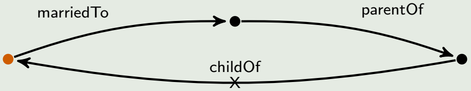
Celebrity

$\text{I}.\forall\text{meets}.\exists\text{knows}.\text{me}$



Stepparent

$\text{I}.\exists\text{marriedTo}.\exists\text{parentOf}.\neg\exists\text{childOf}.\text{me}$



[Marx]

$\mathcal{ALCQ}\mathsf{me}_2$

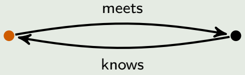
- I remembers the current point of evaluation.
- me only matches the point marked by I.
- There are only 2 or less $\leq_n$, $\exists$, $\forall$ between I and me.

Narcissist

I.∃loves.me

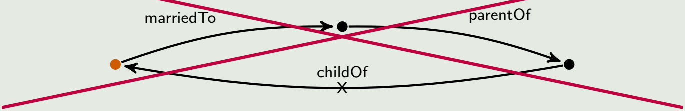
Celebrity

I.∀meets.∃knows.me



Stepparent

~~I.∃marriedTo.∃parentOf.¬∃childOf.me~~



[Marx]

Inconsistent Example

A barber is someone who shaves all people that do not shave themself. Now consider an island on which exactly one barber lives.

Definition

Given an ontology O (ABox, TBox, HBox):

The Ontology O is *consistent*

$$:\Leftrightarrow$$

There is a model for the ontology O

$$\Leftrightarrow$$

There is a non-empty model for O, so that:

1. concepts are represented by sets
2. roles are represented by relations
3. every axiom is hold.

Obvious application

I just created or modified an ontology. Does it make sense?

$\Leftrightarrow$ Is the ontology consistent?

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TBox-Axiom inference

Is a TBox axiom X the inference of an existing consistent TBox T?

$\Leftrightarrow$ Is the TBox $T' = T \cup \{\neg X\}$ still consistent?

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Classification

What is the tree of concepts in a TBox T?

$\Leftrightarrow$ For which pair of concepts C_1, C_2 is $C_1 \sqsubseteq C_2$ a inference of the TBox T?

Consistency checking for $\mathcal{ALCHIQ}$ ontologies

- Problem is decidable (EXPTIME-complete),.
- Consistency checking for a single concept is PSPACE-complete
- Reasoning software already exists (Hermit, Pellet, ...).

Consistency Checking for $\mathcal{ALCQ}me_2$ ontologies

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$\Rightarrow$ Reduce the TBox satisfiability problem over $\mathcal{ALCQ}me_2$ to the one over $\mathcal{ALCHIQ}$polynomially.
$\Rightarrow$ $\mathcal{ALCQ}me_2$ has the same complexity.

[Gorín and Schröder]

Quasi-tree model property

If C is a satisfiable concept of a $\mathcal{ALCQ}$me$_2$-TBox T, then there exists a quasi-tree model that satisfies C at its root.

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Idea: Encode it as a tree

- Force the model to be a tree
- Encode self loops of R as new concepts $\circlearrowleft_R$
- Encode uplinks of R as new concepts $\uparrow_R$
- Encode the behaviour of $\geq, \sqcup, \sqcap, \neg$ as axioms in $\mathcal{ALCHIQ}$

[Gorín and Schröder]

Concept encoding

Semantics

$$I, x, y \models B \text{ iff } y \in B^I \text{ if B is atomic}$$
$$I, x, y \models \neg C \text{ iff } I, x, y \not\models C$$

Encode each original concept (or subformula) C as:

$A_{*,C}$ The closed concept C holds here.

Enforce the $A_{*,C}$ concept to behave like the concept C:

$$\top \sqsubseteq \neg(A_{*,C} \sqcap A_{*,\neg C}),$$
$$\top \sqsubseteq A_{*,\top},$$
$$A_{*,C \sqcap D} \sqsubseteq A_{*,C} \sqcap A_{*,D}$$

Concept encoding

Semantics

$$I, x \vDash A_{f*,C} \text{ iff } I, \text{father of } x, x \vDash C$$
$$I, x \vDash A_{**,C} \text{ iff } I, x, x \vDash C$$
$$I, x \vDash A_{*f,C} \text{ iff } I, x, \text{father of } x \vDash C$$

- $A_{**,C}$ I describes this node and the concept C holds here.
- $A_{f*,C}$ I describes the father node and the concept C holds here.
- $A_{*f,C}$ I describes the this node and the concept C holds at the father node.

Concept encoding

Semantics

$I, x, y \models \text{me}$ iff $x = y$

$I, x, y \models \exists R.C$ iff there is a $z : (y, z) \in R^I$ and $I, x, z \models C$

$I, x, y \models I.C$ iff $I, y, y \models C$

Is encoded as:

$$A_{*,I.\geq_n R.C} \sqsubseteq$$

$$\bigsqcap \left(\begin{array}{r} (\uparrow_R \sqcap A_{*f,C(\bot)}) \sqcap (\circlearrowleft_R \sqcap A_{**,C(\top)}) \rightarrow \geq_{n-2} R.A_{f*,C(\bot)} \\ (\uparrow_R \sqcap A_{*f,C(\bot)}) \sqcap \neg(\circlearrowleft_R \sqcap A_{**,C(\top)}) \rightarrow \geq_{n-1} R.A_{f*,C(\bot)} \\ \neg(\uparrow_R \sqcap A_{*f,C(\bot)}) \sqcap (\circlearrowleft_R \sqcap A_{**,C(\top)}) \rightarrow \geq_{n-1} R.A_{f*,C(\bot)} \\ \neg(\uparrow_R \sqcap A_{*f,C(\bot)}) \sqcap \neg(\circlearrowleft_R \sqcap A_{**,C(\top)}) \rightarrow \geq_{n-0} R.A_{f*,C(\bot)} \end{array} \right)$$

And many more...

Implementation outlook

- Use OWL files for ontologies
- Encode the *I* and *me* as a Role *I* and *me* as concept
- Encode all the OWL axioms (Role hierarchies, Role inverses, ObjectOneOf, ...)
- Check how big the polynomial blow-up
  - of the file-size
  - of consistency checking
  - of classification

  actually is for large ontologies.

- Krötzsch M.; Simancik F.; Horrocks I.: „A Description Logic Primer",
  arXiv:1201.4089v1 [cs.AI]
- Gorín D.; Schröder L.: „Celebrities don't follow their followers: Binding and Qualified Number Restrictions"
- Maarten Marx: „Narcissists, Stepmothers and Spies ", 2002