Googles PageRank-Algorithmus

```
with(LinearAlgebra): with(GraphTheory): with(RandomGraphs):
```

Transformationen zur Konstruktion der Google-Matrix

```
AtoB := proc(A)
 local B, k, rs, i, j;
 k := RowDimension(A);
 B := Matrix(k, k);
 for i from 1 to k do
 rs := add(A[i,j], j = 1..k);
 if rs > 0 then
 for j from 1 to k do B[i,j] := A[i,j]/rs od;
 fi;
 od;
 B
 end:

BtoC := proc(B)
 local C, k, rs, i, j;
 k := RowDimension(B);
 C := Matrix(k, k);
 C := B;
for i from 1 to k do
 rs := add(C[i,j], j = 1..k);
 if rs = 0 then
 for j from 1 to k do C[i,j] := 1/k od;
fi;
 od;
 C
 end:

CtoG := proc(C, β)
 local G, k, i, j;
 k := RowDimension(C);
 G := Matrix(k, k);
 for i from 1 to k do
 for j from 1 to k do
 G[i,j] := β·C[i,j] + (1 − β)/k
 od;
 od;
 G;
 end:
```

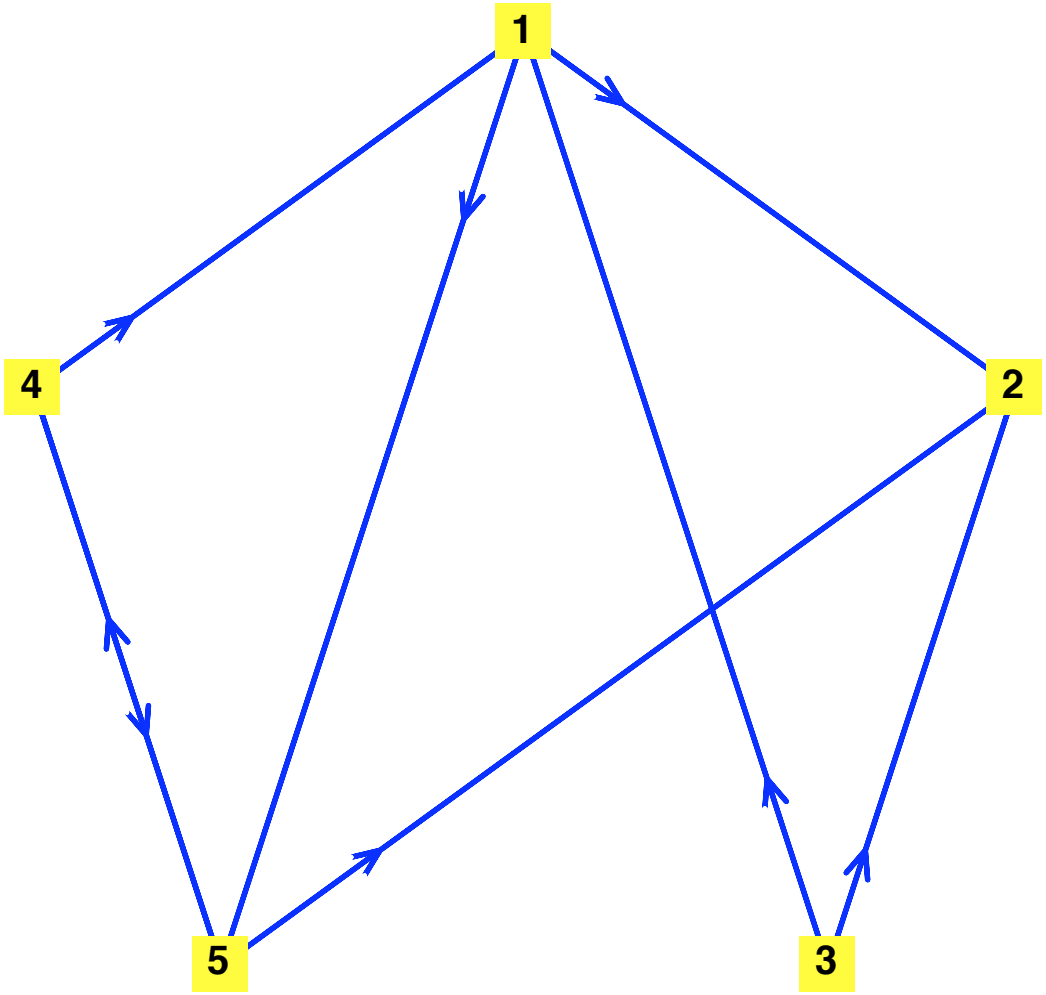
```
AtoG := proc(A, β)
 CtoG(BtoC(AtoB(A)), β)
 end:

normalize := proc(v)
 local s;
 s := convert(v, `+`);
 map(x→ℜ(x/s), v);
 end:
```

1. Beispiel

$W := RandomDigraph(5, 8)$

Graph 4: a directed unweighted graph with 5 vertices and 8 arc(s) **(2.1)**

$DrawGraph(W)$



$A := AdjacencyMatrix(W)$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \textbf{(2.2)}$$

$B := AtoB(A)$

$$\begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix} \tag{2.3}$$

$C := BtoC(B)$

$$\begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix} \tag{2.4}$$

$G := CtoG\left(C, \frac{1}{3}\right);$

$$\begin{bmatrix} \frac{2}{15} & \frac{3}{10} & \frac{2}{15} & \frac{2}{15} & \frac{3}{10} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{3}{10} & \frac{3}{10} & \frac{2}{15} & \frac{2}{15} & \frac{2}{15} \\ \frac{3}{10} & \frac{2}{15} & \frac{2}{15} & \frac{2}{15} & \frac{3}{10} \\ \frac{2}{15} & \frac{3}{10} & \frac{2}{15} & \frac{3}{10} & \frac{2}{15} \end{bmatrix} \tag{2.5}$$

$$G^{10}$$

$$\left[\left[\frac{13482149591597}{65610000000000}, \frac{12037634565439}{49207500000000}, \frac{44181054196229}{295245000000000}, \frac{106934934622}{576650390625}, \frac{126835557444973}{590490000000000}\right], \left[\frac{20223226045237}{98415000000000}, \frac{1504704255437}{6150937500000}, \frac{2454502991767}{16402500000000}, \frac{3041704636547}{16402500000000}, \frac{9665871101}{45000000000}\right], \left[\frac{60669676784249}{295245000000000}, \frac{48150537529381}{196830000000000}, \frac{44181052876729}{295245000000000}, \frac{12166818202867}{65610000000000}, \frac{21139260710683}{98415000000000}\right], \left[\frac{40446449929921}{196830000000000}, \frac{144451616507383}{590490000000000}, \frac{818167639381}{5467500000000}, \frac{109501365756343}{590490000000000}, \frac{126835562893363}{590490000000000}\right], \left[\frac{10111613255779}{49207500000000}, \frac{36112901059967}{147622500000000}, \frac{22090527317827}{147622500000000}, \frac{109501367112653}{590490000000000}, \frac{126835560306823}{590490000000000}\right]\right] \tag{2.6}$$

$$map\left(evalf, G^{10}, 5\right)$$

$$\begin{bmatrix} 0.20547 & 0.24461 & 0.14963 & 0.18543 & 0.21478 \\ 0.20548 & 0.24461 & 0.14963 & 0.18543 & 0.21478 \\ 0.20547 & 0.24461 & 0.14963 & 0.18543 & 0.21478 \\ 0.20547 & 0.24461 & 0.14963 & 0.18543 & 0.21478 \\ 0.20547 & 0.24461 & 0.14963 & 0.18543 & 0.21478 \end{bmatrix} \tag{2.7}$$

$G := evalf(G, 5)$

$$\begin{bmatrix} 0.13333 & 0.30000 & 0.13333 & 0.13333 & 0.30000 \\ 0.20000 & 0.20000 & 0.20000 & 0.20000 & 0.20000 \\ 0.30000 & 0.30000 & 0.13333 & 0.13333 & 0.13333 \\ 0.30000 & 0.13333 & 0.13333 & 0.13333 & 0.30000 \\ 0.13333 & 0.30000 & 0.13333 & 0.30000 & 0.13333 \end{bmatrix} \quad \textbf{(2.8)}$$

$EV := [Eigenvectors(Transpose(G))]:$
$map(evalf, op(1, EV), 5)$

$$\begin{bmatrix} 0.99999 + 0.\,I \\ -0.10690 + 0.12733\,I \\ -0.10690 - 0.12733\,I \\ -0.13537 + 0.\,I \\ 0.082493 + 0.\,I \end{bmatrix} \quad \textbf{(2.9)}$$

$map(evalf, op(2, EV)[1..5, 1], 5)$

$$\begin{bmatrix} -0.45386 + 0.\,I \\ -0.54031 + 0.\,I \\ -0.33051 + 0.\,I \\ -0.40958 + 0.\,I \\ -0.47442 + 0.\,I \end{bmatrix} \quad \textbf{(2.10)}$$

$Transpose(normalize(\%))$

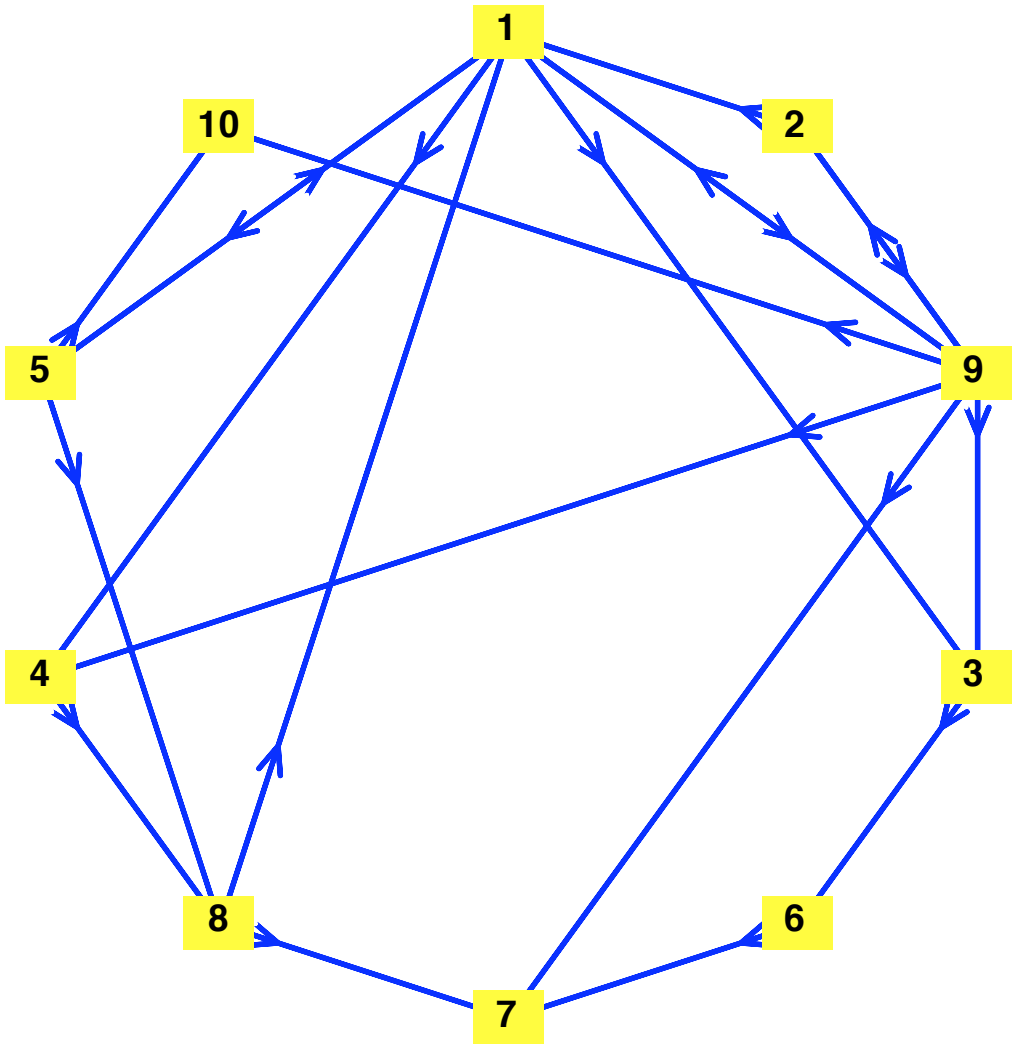
$$\begin{bmatrix} 0.2054892515 & 0.2446302769 & 0.1496414148 & 0.1854410779 & 0.2147979789 \end{bmatrix} \quad \textbf{(2.11)}$$

2. Beispiel

$W := RandomDigraph(10, 20)$

Graph 5: a directed unweighted graph with 10 vertices and 20 arc(s) **(3.1)**

$DrawGraph(W)$



$A := AdjacencyMatrix(W)$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(3.2)}$$

```
G := AtoG(A, 0.2):
map(evalf, G, 5)
```

(3.3)

```
[[0.080000, 0.080000, 0.13000, 0.13000, 0.13000, 0.080000, 0.080000, 0.080000,
  0.13000, 0.080000],
 [0.18000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000,
  0.18000, 0.080000],
 [0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.28000, 0.080000, 0.080000,
  0.080000, 0.080000],
 [0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.28000,
  0.080000, 0.080000],
 [0.14667, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.14667,
  0.080000, 0.14667],
 [0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.28000, 0.080000,
  0.080000, 0.080000],
 [0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000,
  0.10000],
 [0.18000, 0.080000, 0.080000, 0.080000, 0.080000, 0.080000, 0.18000, 0.080000,
  0.080000, 0.080000],
 [0.11333, 0.11333, 0.11333, 0.11333, 0.080000, 0.080000, 0.11333, 0.080000,
  0.080000, 0.11333],
 [0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000, 0.10000,
  0.10000]]
```

```
map(evalf, G^20, 5)
```

(3.4)

```
[[0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
 [0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888,
  0.098664, 0.093534],
```

[0.11318, 0.087540, 0.093198, 0.093198, 0.089910, 0.10289, 0.11901, 0.10888, 0.098664, 0.093534]]

$$EV := [Eigenvectors(Transpose(G))]:$$

$$map(evalf, op(1, EV), 5)$$

$$\begin{bmatrix} 1.0000 + 0.\,I \\ -0.071290 + 0.089992\,I \\ -0.071290 - 0.089992\,I \\ 0.019030 + 0.050269\,I \\ 0.019030 - 0.050269\,I \\ 0.042617 + 0.\,I \\ -0.049049 + 0.018241\,I \\ -0.049049 - 0.018241\,I \\ 2.8464\,10^{-10} + 0.\,I \\ -2.8464\,10^{-10} + 0.\,I \end{bmatrix} \quad \textbf{(3.5)}$$

$$map(evalf, op(2, EV)\ [1..10, 1], 5)$$

$$\begin{bmatrix} -0.35609 + 0.\,I \\ -0.27543 + 0.\,I \\ -0.29323 + 0.\,I \\ -0.29323 + 0.\,I \\ -0.28289 + 0.\,I \\ -0.32373 + 0.\,I \\ -0.37443 + 0.\,I \\ -0.34259 + 0.\,I \\ -0.31043 + 0.\,I \\ -0.29429 + 0.\,I \end{bmatrix} \quad \textbf{(3.6)}$$

$$Transpose(map(evalf, normalize(\%), 5))$$

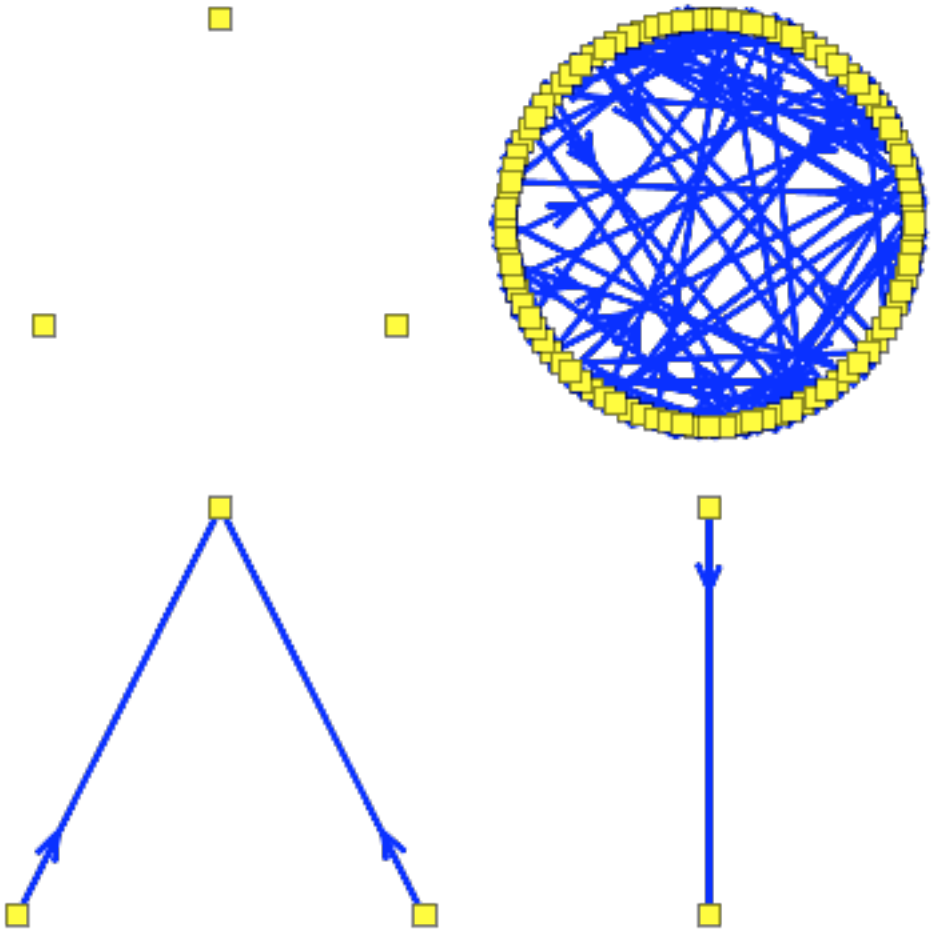
[0.11318, 0.087540, 0.093197, 0.093197, 0.089911, 0.10289, 0.11900, 0.10889, 0.098664, 0.093534] **(3.7)**

3. Beispiel

$W := RandomDigraph(100, 150)$

Graph 21: a directed unweighted graph with 100 vertices and 150 arc(s) **(4.1)**

$DrawGraph(W)$



$A := AdjacencyMatrix(W)$

$$\begin{bmatrix} 100 \text{ x } 100 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: sparse} \\ \text{Order: C\_order} \end{bmatrix} \quad \textbf{(4.2)}$$

$G := AtoG(A, 0.15)$

$$\begin{bmatrix} 100 \text{ x } 100 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(4.3)}$$

$Giter := map(evalf, G^{20}, 3)$

$$\begin{bmatrix} 100 \text{ x } 100 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(4.4)}$$

$convert(Giter[1, 1..100], list)$

$$[0.00950, 0.0100, 0.0102, 0.0107, 0.00982, 0.00881, 0.0101, 0.0106, 0.00939, 0.00933, 0.0109, 0.00881, 0.00881, 0.0100, 0.00909, 0.0104, 0.0115, 0.0101, 0.0113, 0.0116, 0.0122, 0.0103, 0.0105, 0.00881, 0.0103, 0.0100, 0.0129, 0.00881, 0.0102, 0.0105, 0.0110, 0.00929, 0.0107, 0.00881, 0.00909, 0.00881, 0.0101, 0.0103, 0.0110, 0.0117, 0.0115, 0.0103, 0.00881, 0.00997, 0.0106, 0.00881, 0.00956, 0.0105, 0.00971, 0.00939, 0.0107, 0.00994, 0.0102, 0.00991, 0.00881, 0.00957, 0.00881, 0.0100, 0.00881, 0.00916, 0.0104, 0.0103, 0.0116, 0.00881, 0.00911, 0.0124, 0.0104, 0.0101, 0.0110, 0.0111, 0.0113, 0.00911, 0.0107, 0.00881, 0.00956, 0.0101, 0.00881, 0.00966, 0.0122, 0.00881, 0.00881, 0.00917, 0.0108, 0.0105, 0.00881, 0.00916, 0.00935, 0.0108, 0.0120, 0.0106, 0.0102, 0.0101, 0.00881, 0.00881, 0.0101, 0.00881, 0.00881, 0.0102, 0.00951, 0.00881] \quad \textbf{(4.5)}$$

convert(*Giter*[55, 1 ..100], *list*)

[0.00950, 0.0100, 0.0102, 0.0107, 0.00982, 0.00881, 0.0101, 0.0106, 0.00939, 0.00933, 0.0109, 0.00881, 0.00881, 0.0100, 0.00909, 0.0104, 0.0115, 0.0101, 0.0113, 0.0116, 0.0122, 0.0103, 0.0105, 0.00881, 0.0103, 0.0100, 0.0129, 0.00881, 0.0102, 0.0105, 0.0110, 0.00929, 0.0107, 0.00881, 0.00909, 0.00881, 0.0101, 0.0103, 0.0110, 0.0117, 0.0115, 0.0103, 0.00881, 0.00997, 0.0106, 0.00881, 0.00956, 0.0105, 0.00971, 0.00939, 0.0107, 0.00994, 0.0102, 0.00991, 0.00881, 0.00957, 0.00881, 0.0100, 0.00881, 0.00916, 0.0104, 0.0103, 0.0116, 0.00881, 0.00911, 0.0124, 0.0104, 0.0101, 0.0110, 0.0111, 0.0113, 0.00911, 0.0107, 0.00881, 0.00956, 0.0101, 0.00881, 0.00966, 0.0122, 0.00881, 0.00881, 0.00917, 0.0108, 0.0105, 0.00881, 0.00916, 0.00935, 0.0108, 0.0120, 0.0106, 0.0102, 0.0101, 0.00881, 0.00881, 0.0101, 0.00881, 0.00881, 0.0102, 0.00951, 0.00881] **(4.6)**

convert(*Giter*[88, 1 ..100], *list*)

[0.00950, 0.0100, 0.0102, 0.0107, 0.00982, 0.00881, 0.0101, 0.0106, 0.00939, 0.00933, 0.0109, 0.00881, 0.00881, 0.0100, 0.00909, 0.0104, 0.0115, 0.0101, 0.0113, 0.0116, 0.0122, 0.0103, 0.0105, 0.00881, 0.0103, 0.0100, 0.0129, 0.00881, 0.0102, 0.0105, 0.0110, 0.00929, 0.0107, 0.00881, 0.00909, 0.00881, 0.0101, 0.0103, 0.0110, 0.0117, 0.0115, 0.0103, 0.00881, 0.00997, 0.0106, 0.00881, 0.00956, 0.0105, 0.00971, 0.00939, 0.0107, 0.00994, 0.0102, 0.00991, 0.00881, 0.00957, 0.00881, 0.0100, 0.00881, 0.00916, 0.0104, 0.0103, 0.0116, 0.00881, 0.00911, 0.0124, 0.0104, 0.0101, 0.0110, 0.0111, 0.0113, 0.00911, 0.0107, 0.00881, 0.00956, 0.0101, 0.00881, 0.00966, 0.0122, 0.00881, 0.00881, 0.00917, 0.0108, 0.0105, 0.00881, 0.00916, 0.00935, 0.0108, 0.0120, 0.0106, 0.0102, 0.0101, 0.00881, 0.00881, 0.0101, 0.00881, 0.00881, 0.0102, 0.00951, 0.00881] **(4.7)**

$EV := [\,Eigenvectors(\,Transpose(\,G\,)\,)\,];$

$$\left[\begin{bmatrix} 1\,..\,100\ Vector_{column} \\ Data\ Type:\ complex_8 \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix}, \begin{bmatrix} 100\ x\ 100\ Matrix \\ Data\ Type:\ complex_8 \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix}\right] \tag{4.8}$$

$op(\,1, EV\,)\ [\,1\,]$

$$0.99999999999999845 + 0.\,I \tag{4.9}$$

$eigenV := op(\,2, EV\,)\ [\,1\,..100, 1\,]$

$$\begin{bmatrix} 1\,..\,100\ Vector_{column} \\ Data\ Type:\ complex_8 \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix} \tag{4.10}$$

$convert(\,map(\,evalf, eigenV, 3\,), list\,)$

$$\begin{aligned}[\,&-0.0945 + 0.\,I, -0.100 + 0.\,I, -0.101 + 0.\,I, -0.106 + 0.\,I, -0.0978 + 0.\,I, -0.0877 \\ &+ 0.\,I, -0.101 + 0.\,I, -0.106 + 0.\,I, -0.0934 + 0.\,I, -0.0929 + 0.\,I, -0.108 + 0.\,I, \\ &-0.0877 + 0.\,I, -0.0877 + 0.\,I, -0.0995 + 0.\,I, -0.0904 + 0.\,I, -0.104 + 0.\,I, \\ &-0.114 + 0.\,I, -0.101 + 0.\,I, -0.112 + 0.\,I, -0.116 + 0.\,I, -0.122 + 0.\,I, -0.103 \\ &+ 0.\,I, -0.104 + 0.\,I, -0.0877 + 0.\,I, -0.103 + 0.\,I, -0.0997 + 0.\,I, -0.128 + 0.\,I, \\ &-0.0877 + 0.\,I, -0.102 + 0.\,I, -0.104 + 0.\,I, -0.110 + 0.\,I, -0.0924 + 0.\,I, -0.106 \\ &+ 0.\,I, -0.0877 + 0.\,I, -0.0904 + 0.\,I, -0.0877 + 0.\,I, -0.101 + 0.\,I, -0.102 \\ &+ 0.\,I, -0.110 + 0.\,I, -0.117 + 0.\,I, -0.114 + 0.\,I, -0.103 + 0.\,I, -0.0877 + 0.\,I, \\ &-0.0993 + 0.\,I, -0.106 + 0.\,I, -0.0877 + 0.\,I, -0.0951 + 0.\,I, -0.104 + 0.\,I, \\ &-0.0966 + 0.\,I, -0.0934 + 0.\,I, -0.107 + 0.\,I, -0.0990 + 0.\,I, -0.101 + 0.\,I, \\ &-0.0986 + 0.\,I, -0.0877 + 0.\,I, -0.0952 + 0.\,I, -0.0877 + 0.\,I, -0.0998 + 0.\,I, \\ &-0.0877 + 0.\,I, -0.0912 + 0.\,I, -0.104 + 0.\,I, -0.103 + 0.\,I, -0.116 + 0.\,I, \\ &-0.0877 + 0.\,I, -0.0907 + 0.\,I, -0.123 + 0.\,I, -0.103 + 0.\,I, -0.100 + 0.\,I, -0.109 \\ &+ 0.\,I, -0.110 + 0.\,I, -0.112 + 0.\,I, -0.0907 + 0.\,I, -0.106 + 0.\,I, -0.0877 + 0.\,I, \\ &-0.0952 + 0.\,I, -0.100 + 0.\,I, -0.0877 + 0.\,I, -0.0961 + 0.\,I, -0.121 + 0.\,I, \\ &-0.0877 + 0.\,I, -0.0877 + 0.\,I, -0.0913 + 0.\,I, -0.108 + 0.\,I, -0.104 + 0.\,I, \\ &-0.0877 + 0.\,I, -0.0912 + 0.\,I, -0.0930 + 0.\,I, -0.107 + 0.\,I, -0.119 + 0.\,I, \\ &-0.105 + 0.\,I, -0.101 + 0.\,I, -0.101 + 0.\,I, -0.0877 + 0.\,I, -0.0877 + 0.\,I, -0.101 \\ &+ 0.\,I, -0.0877 + 0.\,I, -0.0877 + 0.\,I, -0.102 + 0.\,I, -0.0946 + 0.\,I, -0.0877 \\ &+ 0.\,I\,] \end{aligned} \tag{4.11}$$

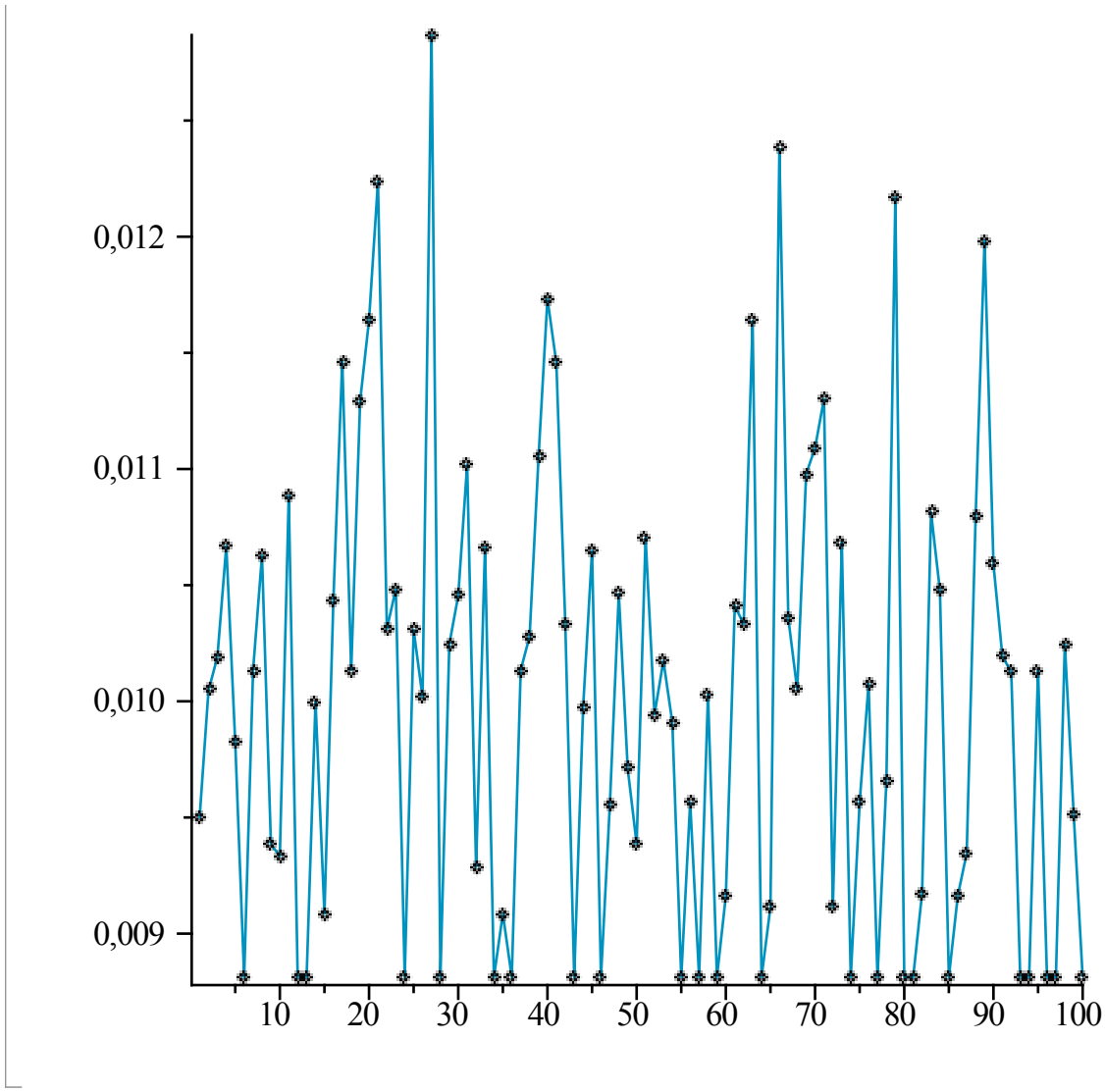
```
Transpose(map(evalf, normalize(eigenV), 5))
```

$$\begin{bmatrix} 1 .. 100\ Vector_{row} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix} \tag{4.12}$$

```
L := convert(%, list)
```

[0.0094959, 0.010046, 0.010191, 0.010669, 0.0098232, 0.0088123, 0.010134, 0.010628, 0.0093855, 0.0093329, 0.010886, 0.0088123, 0.0088123, 0.0099986, 0.0090857, 0.010432, 0.011465, 0.010134, 0.011292, 0.011642, 0.012238, 0.010312, 0.010483, 0.0088123, 0.010308, 0.010017, 0.012868, 0.0088123, 0.010246, 0.010455, 0.011020, 0.0092871, 0.010665, 0.0088123, 0.0090857, 0.0088123, 0.010134, 0.010280, 0.011050, 0.011733, 0.011456, 0.010335, 0.0088123, 0.0099733, 0.010648, 0.0088123, 0.0095580, 0.010463, 0.0097097, 0.0093855, 0.010705, 0.0099430, 0.010175, 0.0099087, 0.0088123, 0.0095658, 0.0088123, 0.010031, 0.0088123, 0.0091628, 0.010412, 0.010332, 0.011638, 0.0088123, 0.0091145, 0.012378, 0.010358, 0.010050, 0.010978, 0.011091, 0.011302, 0.0091145, 0.010685, 0.0088123, 0.0095646, 0.010075, 0.0088123, 0.0096592, 0.012171, 0.0088123, 0.0088123, 0.0091745, 0.010814, 0.010476, 0.0088123, 0.0091628, 0.0093465, 0.010795, 0.011982, 0.010589, 0.010194, 0.010125, 0.0088123, 0.0088123, 0.010134, 0.0088123, 0.0088123, 0.010247, 0.0095088, 0.0088123] **(4.13)**

```
with(plots) : with(Statistics) :
LineChart(L)
```


0,012
0,011
0,010
0,009
10
20
30
40
50
60
70
80
90
100